

# DYNAMICAL ANALYSIS OF THE DEFORMABLE POWER TRANSMISSION MECHANISM USING MIXED FEM AND MBS METHOD

Małgorzata Zglińska, Grzegorz Orzechowski, Janusz Frączek

Warsaw University of Technology  
Institute of Aeronautics and Applied Mechanics  
Nowowiejska Street 24, 00-665 Warsaw, Poland  
e-mail: jfraczek@meil.pw.edu.pl

## Abstract

*In the article an algorithm of dynamic analysis of mechanisms of industrial robots is presented using mixed FEM and MBS method. In simulation analysis of this kind of constructions finite elements method (FEM) is commonly used. An alternative method is mixed multibody system (MBS) and FEM formulation. Despite the fact that this formulation undergoes a few limitations, it is often best choice between accuracy and computation efficiency.*

*Mechanisms analyzed in the paper are manipulators of parallel structures. Friction forces in joints are included in the model. A sequence of simulations of the robot behaviour was performed with different input functions (driving torques). Finally the discussion of influence of different integration methods on the accuracy and efficiency of numerical calculations is presented.*

*The paper presents among other schema of modelling with FEM and MBS formulation, CAD model of the parallel Cartesian robot, Manipulator model in MSC.ADAMS application, graph of the driving force and tip orientation differences model of the body with visible parallelograms, FEM models, MBS model of POLYCRANK robot power transmission mechanism, modal shapes corresponded to the lowest modal frequency, driving torque vs. time, angular velocity of the upper shield, reaction forces in joint between cap and disc.*

**Keywords:** dynamic analysis, robots, FEM, MBS, multibody systems

## 1. Preface

In the industrial robots design process an estimation of dynamical properties of robot mechanisms is usually required. Especially interesting is information concerning positioning accuracy, range of movement, workspace, strength characteristics and general behaviour of robot under large dynamic loads.

In simulation analysis of those systems which take into account flexibility of robot arms, the most commonly used is finite elements method (FEM) [2]. However in case of complex mechanisms (large number of degrees of freedom) and in case of large displacement FEM analysis is labour-consuming in model preparation and require time-consuming computations. In addition in many variants of analysis of complex systems difficulties with solution convergence may appear. Moreover consideration of influence of the control system on robot dynamics is rather troublesome.

For this reason in simulation of robot behaviour in dynamic conditions mixed multibody system (MBS) [4, 8] and finite elements method are applied. In this case models of flexible bodies are prepared as substructures in FEM program and then these substructures are used to build the whole model in dedicated MBS program. Kinematical and dynamical analysis of the robot mechanism is performed in MBS package. It should be pointed out that MBS packages have large possibilities to include dynamics of the control systems. Strength (stress) analysis may be performed directly in MBS program or using results obtained from MBS simulations as input data to FEM computations.

However MBS and FEM mixed formulation undergo a few limitations. We usually assume that deformations of bodies are small (although displacements of bodies can be large). Moreover

substructure is represented by dozen or so first (or chosen) modes. In addition FEM model of the substructure cannot be modified during the simulation. In spite of those inconveniences this formulation is often best choice between accuracy and computations efficiency.

In this article we present computation algorithm of dynamical analysis with examples of usage mixed FEM and MBS formulation in modelling of two manipulators constructions, which were designed in the Warsaw University of Technology [5]. Both constructions belong to the class of parallel robots, which have complex structure but higher stiffness and positioning accuracy in comparison to robots with serial structure.

The method of friction modelling in kinematical pairs is also described including initial load of bodies. Influence of the friction and initial load on free vibrations of robots is discussed. Output functions – angular speed and acceleration of chosen bodies, friction forces and reaction forces in kinematics pairs and stress maps in chosen bodies are presented. In addition the discussion of influence of the different integration methods on the accuracy of results is described briefly.

All computations were performed in the environment of computer CAD/CAE packages commonly used in virtual prototyping: UNIGRAPHICS NX [10], ANSYS [1] and MSC.ADAMS [6]. Geometry models of the mechanism were created in the environment of NX program. To build flexible bodies FEM package ANSYS was used. Finally dynamic analysis of mechanisms was performed using MBS package MSC.ADAMS.

## 2. Theoretical background of the dynamic multibody analysis

Most of computations were performed in MSC.ADAMS package, in which the MBS dynamics method is implemented. This method differs from the typical algorithms used in most FEM formulations. Therefore we will shortly describe the theoretical basics of algorithms used in dynamical analysis of rigid and flexible MBS [4, 6].

In classical MBS formulations in absolute coordinates intermediate body-fixed local reference frame body is introduced (called *floating frame*) to describe large motion and small deformations of body. Generalized coordinates representing large motion of bodies are a system of six coordinates which describe movement of local reference frame of the unconstrained body (for example three Cartesian coordinates of the centre of local reference and three Euler angles) with respect to the global (fixed) reference frame. Body deformations are described using FEM linear theory, with respect to the local reference. Equations of motion of the whole system may be written using, for example, Lagrange equations of the first kind in the form:

$$\begin{bmatrix} \mathbf{m}_{ss} & \mathbf{m}_{sf} \\ \mathbf{m}_{fs} & \mathbf{m}_{ff} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_s \\ \ddot{\mathbf{q}}_f \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{ff} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_s \\ \dot{\mathbf{q}}_f \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{q}_f \end{bmatrix} = \begin{bmatrix} (\mathbf{Q}_z)_s \\ (\mathbf{Q}_z)_f \end{bmatrix} + \begin{bmatrix} (\mathbf{Q}_v)_s \\ (\mathbf{Q}_v)_f \end{bmatrix} - \Phi_q^T \lambda \quad (1)$$

where:  $\mathbf{q} = [\mathbf{r}^T, \boldsymbol{\theta}^T, \mathbf{q}_f^T]^T = [\mathbf{q}_s^T, \mathbf{q}_f^T]^T$  – vector of the rigid and flexible general coordinates,  $\mathbf{m}$  – mass matrixes,  $\mathbf{D}$  – dumping matrix (as a result of dissipation function),  $\Phi$  – vector of constraint equations imposed by joints between bodies,  $\Phi_q^T \lambda$  – vector of reaction forces with Lagrange multipliers,  $\mathbf{Q}_z$  – vector of external forces applied to body,  $\mathbf{Q}_v$  – vector of centrifugal, Coriolis and other forces, which are result of differentiation of the kinematic energy with respect to time and coordinates.

System of equations of motion (1) is the system of differential-algebraic equations (DAE) (because solution have to satisfy system of constraint equations, which are algebraic) with large number of degrees of freedom which depends directly on the number of degrees of freedom of the FEM model. That is why in general reduction of degrees of freedom is performed with usage of the modal synthesis algorithm. One of the most common in the structural dynamics is Craig-Bampton method [3]. Using this method system of equations (1) can be represented in the new modal coordinates:

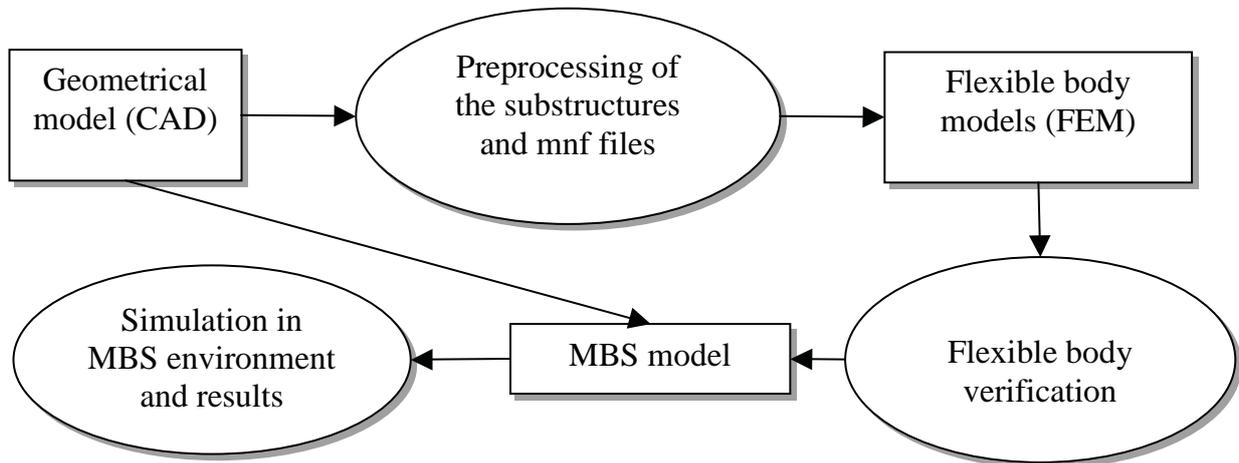
$$\begin{bmatrix} \mathbf{m}_{ss} & \mathbf{m}_{sf} \Psi \\ \Psi^T \mathbf{m}_{fs} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_s \\ \dot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{d} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_s \\ \dot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Omega \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} (\mathbf{Q}_z)_s \\ \Psi^T (\mathbf{Q}_z)_f \end{bmatrix} + \begin{bmatrix} (\mathbf{Q}_v)_s \\ \Psi^T (\mathbf{Q}_v)_f \end{bmatrix} - \begin{bmatrix} \Phi_{q_s}^T \\ \Psi^T \Phi_{q_f}^T \end{bmatrix} \lambda \quad (2)$$

where  $\Psi$  – rectangular transformation matrix,  $\mathbf{p}$  – new sets of coordinates obtained after coordinate transformation,  $\mathbf{I}$  – identity matrix,  $\Omega$  – matrix with the squares of the modal frequencies on diagonal,  $\mathbf{d}$  – matrix of modal dumping.

It should be noticed that:

- A size of the system of equations (2) depends on the number of modal coordinates taken into consideration in transformation matrix  $\Psi$ . From numerical difficulty point of view a size of this matrix should be small; simultaneously this size decides on modal content of solution, so it cannot be chosen in arbitrary manner.
- System (2) is DAE system of equations with high differential index (equal to 3). There are several methods to deal with this kind of system; it should be pointed out that effectiveness of the numerical integration of these systems depends not only on the integration algorithm but also on the form of the system.

From the user point of view modelling process with mixed FEM and MBS formulation is carried out according to the schema shown in the Figure 1.



*Fig. 1. Schema of modelling with mixed FEM and MBS formulation*

As it mentioned earlier general CAD package (NX in this work) can be used to build geometrical models of bodies. Substructures of the flexible bodies should be prepared in FEM application (such as ANSYS), while full model and its analysis should be performed in MBS application (MSC.ADAMS).

In addition after simulation of the MBS model is performed, analysis results may be exported to the FEM application in order to perform more detailed analysis (e.g. fatigue or stress evaluation).

### **3. Dynamic analysis of the prototype of parallel Cartesian manipulator**

Dynamics of the parallel Cartesian manipulator [7] was investigated. The general idea and design of the robot was introduced in thesis [11]. Dynamical analysis of the robot with flexible bodies was performed to investigate general properties of the robot during high speed motion. Moreover the position accuracy was investigated provided that flexibility of robot bodies was taken into account.

In Figure 2 the manipulator model prepared in NX environment is shown. It consists of tree identical assemblies, which are composed of arms (4, 6) and drives (2). Those assemblies are connected at one side to the frame (1), and on the other to the robot tip (8). The bodies of arms are

connected with revolute joints (5). Moreover one of the arms is connected both with driving actuator (3) and with the tip (7) by revolute joint.

Manipulator structure is over-constrained. Theoretical mobility (Grubler count) is 0, but real number of degrees of freedom is equal 3 (the tip of the manipulator can be translated along three axis, without possibility of rotation).

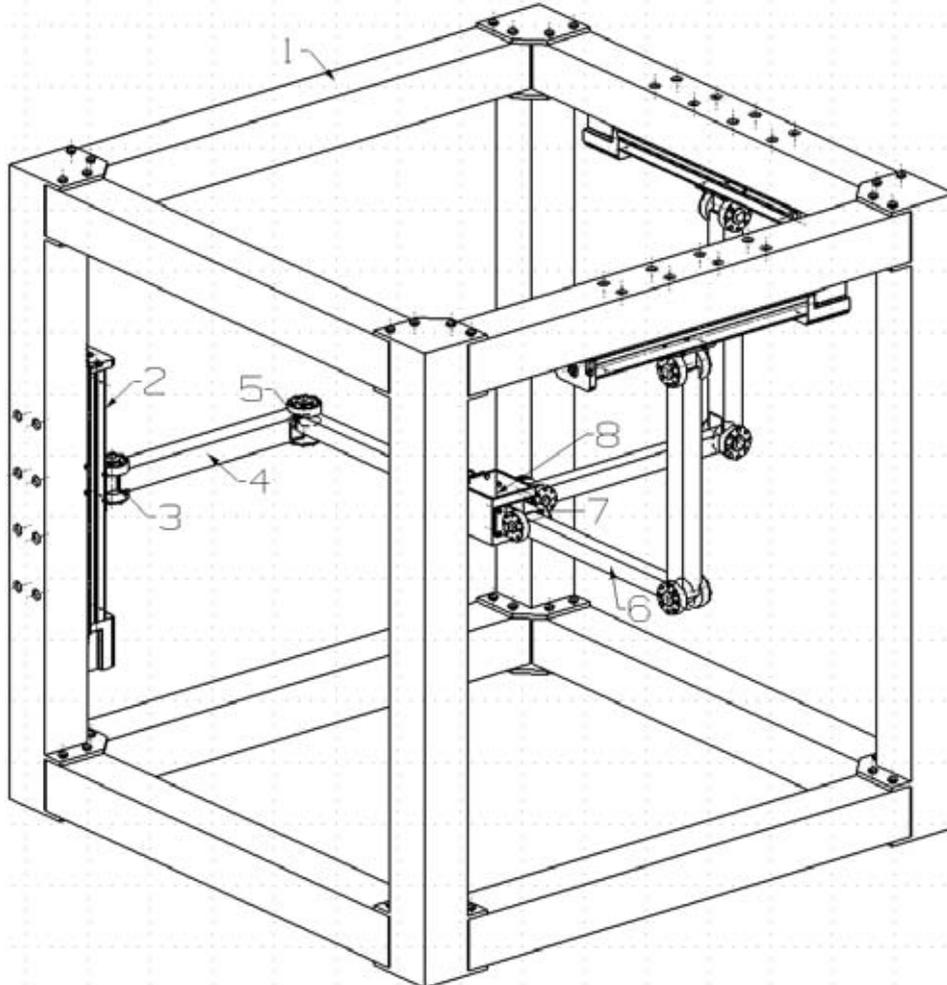


Fig. 2. CAD model of the parallel Cartesian robot

Firstly the manipulator model using CAD package was built. Then models of substructures (robots arms) were prepared in the FEM package and finally sequence of MBS simulations was carried out. Main goal of those simulations was estimation of arms deformability and vibrations analysis under forces driving robot.

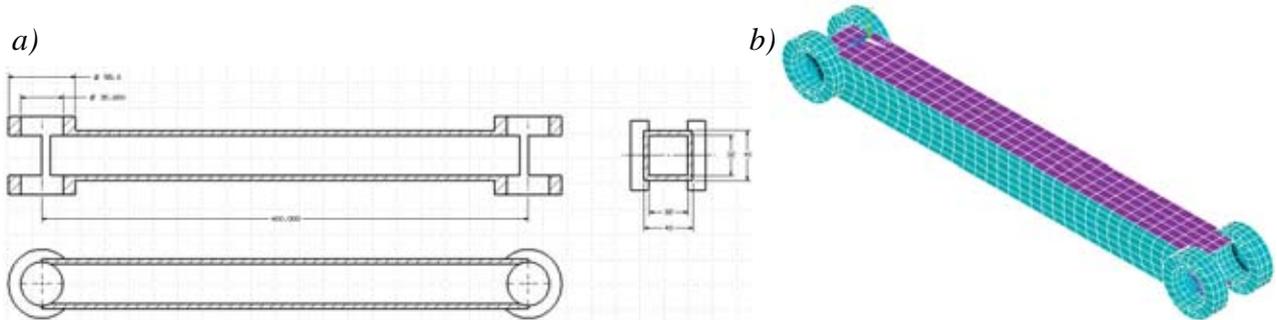
It was assumed that tip trajectory accuracy is influenced mainly by flexibility of manipulators arms. So that those arms were modelled as flexible bodies. Remaining bodies were modelled as rigid. Backlash and friction in joints were neglected.

### 3.1. Models of the flexible bodies

Deformable models of the arms were prepared in dedicated FEM package and then exported as substructures to mnf (modal natural file) file. Preprocessing and substructure analysis was carried out in ANSYS environment. In Figure 3a CAD model of one of arms designed with NX program is shown, while in the Figure 3b meshed model of the same arm is shown.

Middle part of arm was modelled as thin-walled square tube, with SHELL63 elements whereas to build ends of arm SOLID45 elements were applied. It was assumed that robot arms are made of steel.

At both ends robot arms are connected to the rest of model with revolute joints. Therefore in both ends the spider webs of light-weight and stiff (in comparison with the rest of the model) beam elements were introduced (BEAM4) to model connection of the elements of FEM model. Spider webs of beam elements play a role of distributors of loads derived from the rest of model over the whole contact surface.



*Fig. 3. Manipulator arm (4): a) construction draft, b) FEM model*

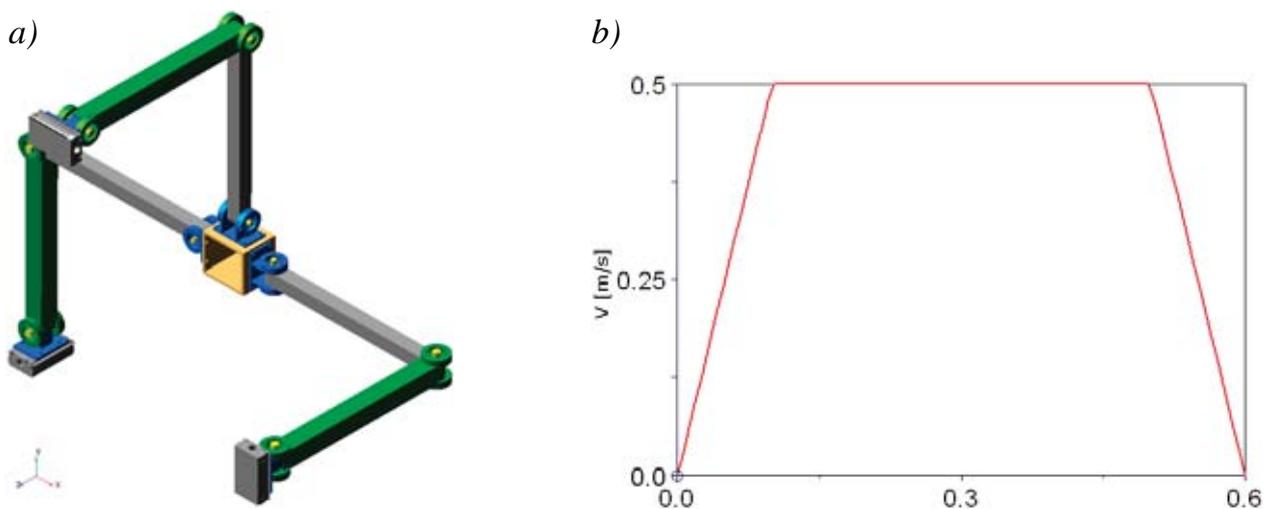
Usually before transforming substructures (mnf files) prepared in FEM packages to the MBS packages, formal verification of substructures is carried out. In this work it was performed in two steps:

- first modal analysis of arm FEM models built in FEM program was performed with some nodes of the substructure fixed (lumped mass approximation was switched on),
- the substructure (as mnf file) was imported to the MBS program, where modal analysis of the linear model was performed with corresponding nodes fixed.

Given frequencies from both modal analyses were compared.

### 3.2. Dynamic analysis of the multibody manipulator model

In the Figure 4a the model of the manipulator built in MSC.ADAMS environment is presented. In this model rigid frame is not shown since it does not influence manipulator dynamic characteristics.



*Fig. 4. a) Manipulator model in MSC.ADAMS application, b) Speed input function in translational joints*

Modal (structural) damping in deformable arms was set to default value fixed by the MSC.ADAMS program. Mechanism motion was defined in the first variant of analysis by

definitions of driving constraints (kinematical analysis) in translational joints (consistently with linear drives). In the second variant of analysis the robot was driven by forces of actuators (dynamics).

In the Fig. 4b trapezoidal speed function is shown, which describes driving constraints in translational joints during simulation with kinematics constraint. In the first simulation only one driving constraint was defined. In the second all three constraints were defined in translational joints.

Next the analysis with force input (driving forces) was performed. Example input force is shown in Figure 5a. The value of the input force was defined as constant in time intervals.

In the Figure 5b the exemplary output functions of the angles determining differences in tip orientation in reference to rigid model are shown. It can be seen that robot is relatively rigid and errors in tip orientation on the dynamic trajectory are relatively small.

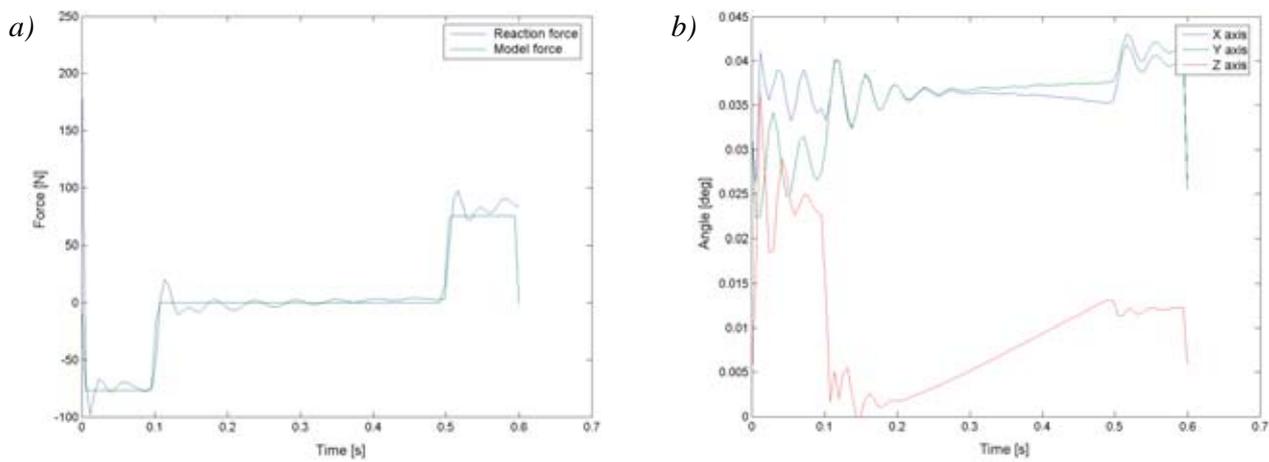


Fig. 5. Graph of the driving force and tip orientation differences in first load variant: a) driving force, b) orientation differences

Results of analyses shown in the Fig. 5b suggest that in some variants of dynamical loads noticeable oscillations may appear. This effect is currently under considerations.

To verify numerically obtained results, several simulations were carried out, with usage of different integration methods with constraint stabilization (SI1 method in comparison to I3).

#### 4. Dynamic analysis of the POLYCRANK robot power transmission mechanisms

Dynamics of the fragment of power transmission mechanism of POLYCRANK robot [12] was investigated. The prototype of robot was built in Warsaw University of Technology [5]. Unique feature of this manipulator is a possibility to perform unlimited rotational motion in almost all articulated joints. Internal driving mechanisms of the robot are made of diagonal cranks. Those cranks have coating construction with composite shields. To transmit power the parallelogram linkages were used – they can transmit power from direct drives. The main parts are shown in the Fig. 6a. Robot has six degrees of freedom and consists of seven bodies.

Complete model of the fragment of geometry of the power transmission mechanism consisting of bodies 4 and 5 (Fig. 6b) was built in NX program environment.

The power transmission mechanisms (bodies 4 and 5) are composed of the following components:

- rotational discs, connected mutually with cross-roller bearings,
- composite shields, which serve as protection of the power transmission mechanism. Those shields also make construction more stiffness,

- parallelogram linkages and additional elements to fix bearings.

This mechanism is strongly over-constrained – theoretical mobility obtained from Grubler count is equal to -111. In fact real number of degrees of freedom is equal 4 – four insides parallelogram linkages (two in body 4 connected with another two in body 5) transmit power to the grip (7), outside parallelogram linkage (on body 4) transmit power to the upper shield (5), while body 4 is driven through applying load to its shield from drive on body 3.

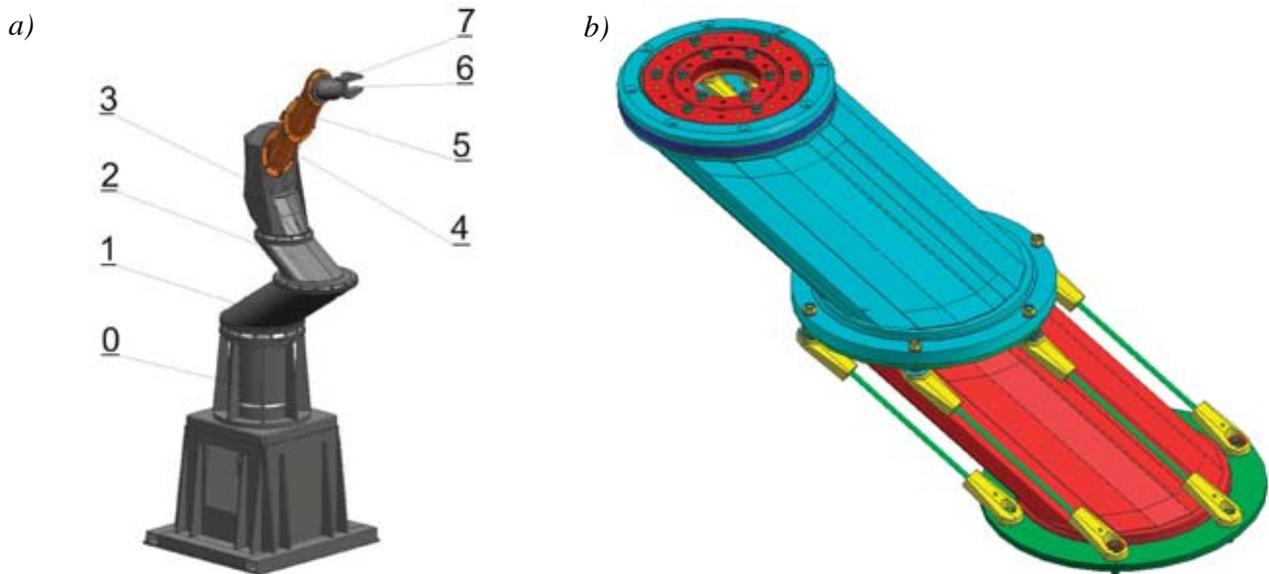


Fig. 6. POLYCRANK robot: a) general view (CAD model in NX program), b) power transmission mechanism (model in ADAMS program)

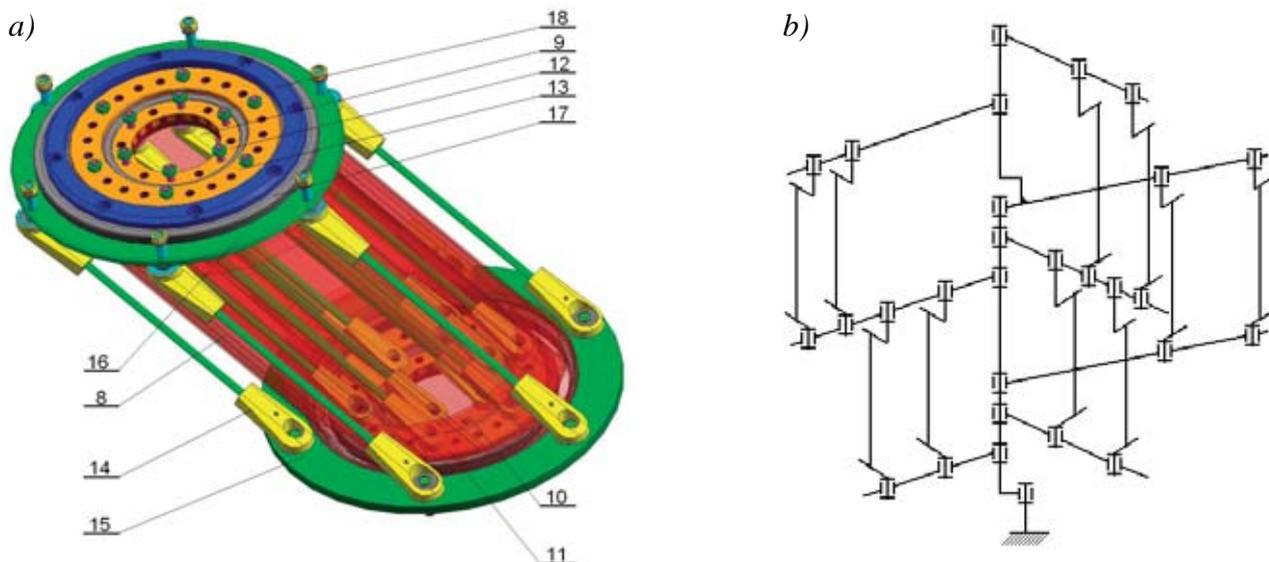


Fig. 7. a) Model of the body 4 with visible parallelograms, b) One of the versions of kinematic schema of power transmission mechanism

Figure 7a illustrates lower body with visible outer parallelogram linkage and two inner parallelogram linkages covered by shield (8). Parallelogram linkages consist of parallel disks (10, 12 or 11, 13 or 15, 16) and links (9, 14) which transmit power between discs. Each parallelogram linkage has six links (each link consist of two heads and one pull rod).

In the Figure 7b one of versions of kinematic schemas is shown. For simplicity only two links of each parallelogram linkage are shown.

The task consisted in preparing the power transmission mechanism MBS model with taking into consideration flexibility of some bodies. Series of dynamic simulations were carried out with considering body load characteristics and body stress for different kind of motion.

It should be pointed out that some simplification was made in MBS model in MSC.ADAMS package. Only pull rods and shields were modelled as flexible. Flexibility of bearings and joints was neglected. The small parts which do not have influence on the dynamics of the robot like screws or washers were eliminated from the model.

#### 4.1. Models of the flexible bodies

Similarly as in the case of analysis of Cartesian manipulator, FEM substructures have been prepared in ANSYS package. First body, which was modelled as flexible, was double-nutted bolt (pull rod). In the Figure 8a FEM model of this body is presented. To model these body elements SOLID45 were applied. At both ends of the pull rod, spider webs were created.

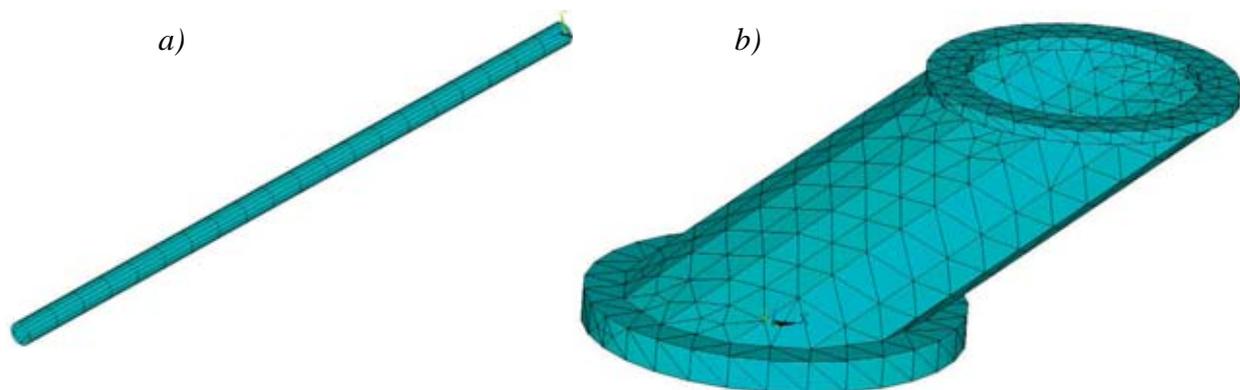


Fig. 8. FEM models: link in parallelograms: a) double-nutted bolt, b) shield of body 5

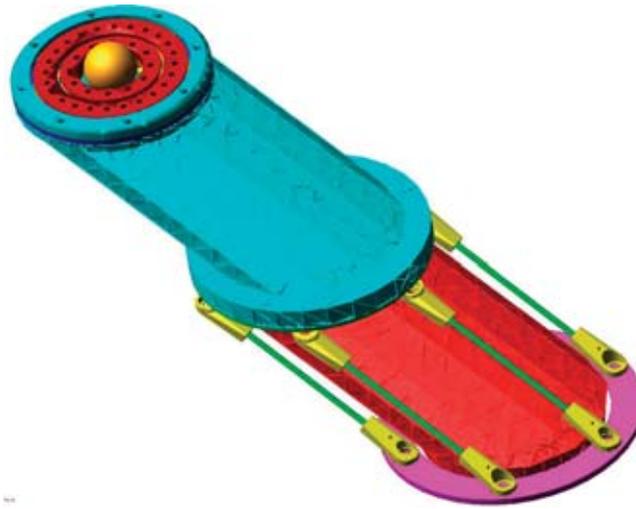
To create flexible bodies of shields, geometry created in NX program was used. The body was meshed automatically using SOLID186 finite elements (Figure 8b). To connect body to other elements (disks) spider webs from beam elements were created. To verify models formally a comparison of modal frequencies calculated in FEM (ANSYS program) and in MBS environment separately was carried out.

In the real prototype of the robot links of parallelogram linkages are mounted with initial load. This effect was taken into consideration in the model by modifications in mnf files. The MBS utility called *mnfload* was used (this application is part of MSC.ADAMS program).

#### 4.2. Dynamic analysis of driving mechanism

Based on kinematic schema (Figure 7b) of the power transmission mechanism all parts in the model were connected by joints. The complete model of the manipulator with flexible pull rods built in MBS program (a view) is shown in the Fig. 9. An additional point mass (5 kg) representing load carried by robot (grip and manipulating object) was placed on the upper disk of the robot. The total weight of the mechanism is equal to 16.25 kg. Several variants of the model were built – with or without friction in bearings included and with initial loads applied or in case when links are not initially loaded.

In the first step modal analysis of the whole manipulator was performed for many different configurations. The level of the lowest frequencies of the mechanism is important information for control system synthesis. During modal analysis all rigid degrees of freedom in mechanism were blocked.



*Fig. 9. MBS model of POLYCRANK robot power transmission mechanism (view)*



*Fig. 10. Modal shapes corresponded to the lowest modal frequency*

On the basis of obtaining results we found out that initial load have a small influence on modal frequencies and modal vectors. In Figure 10 modal vectors corresponded to the first modal frequency (about 111 Hz) are shown – this shape represent bending of the construction along axis perpendicular to the figure.

In the next step several dynamical analyses of the robot mechanism were performed taking into consideration different values of driving torques.

Following results were analyzed:

- angles of revolution, angular velocity and acceleration of the shields,
- reaction forces in joints,
- von Mises reduced stress for pull rods.

For integration of equation of motion Gear algorithm with constraint stabilization (SI1) was used. This algorithm was chosen on the basis of the initial tests, which proved it good numerical efficiency.

In the analysis, different variants of model were also analyzed: one with rigid bodies and four with deformable bodies with or without friction and with or without initial load in pull rods.

In the Figures 12 and 13 some results of the analysis with input driving torque (Fig. 11) are shown. The level of stresses in links is rather small – less then several dozen MPa. Presence of friction and initial loads does not change stresses significantly. However it can be noticed in the Fig. 13 that if we consider friction and initial load reaction force in joints become considerably greater. Moreover it can be seen (Fig. 12) that friction and initial loads cause faster energy loss then in other models.

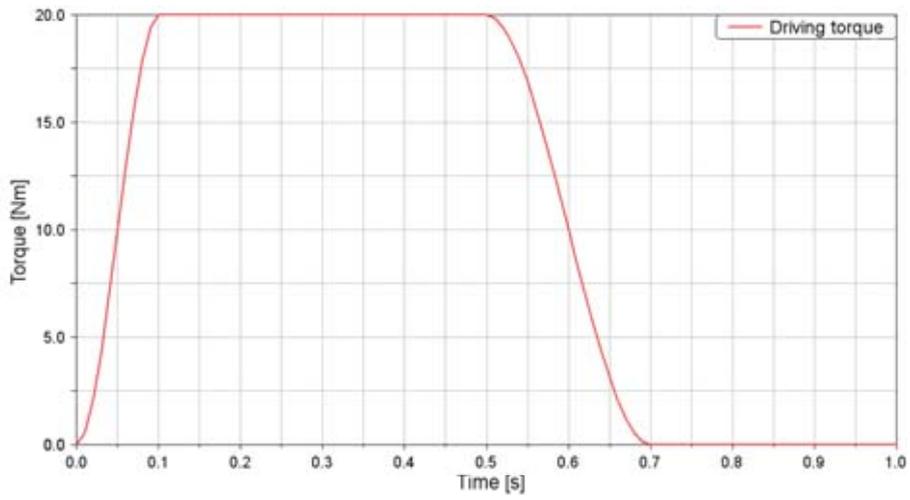


Fig. 11. Driving torque vs. time

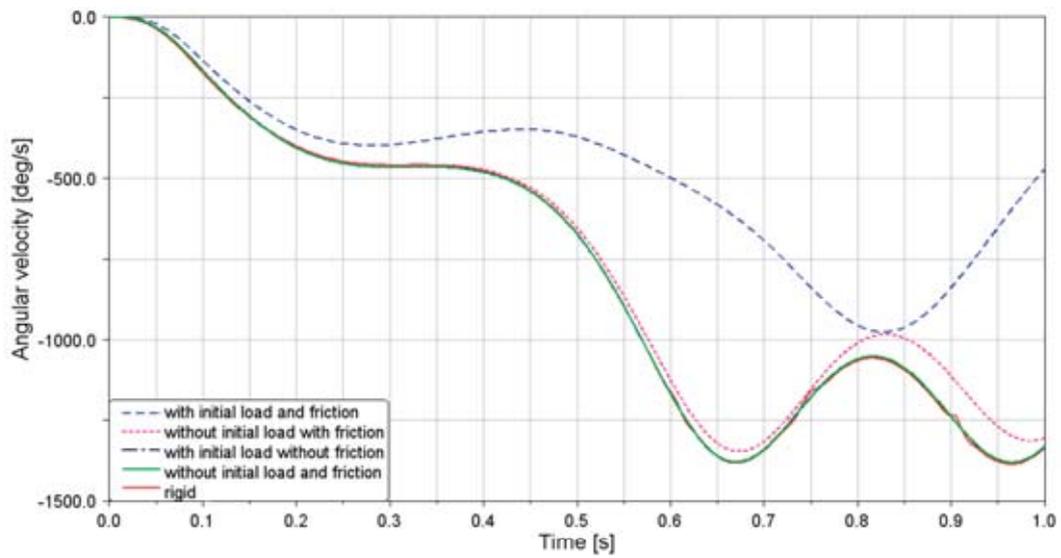


Fig. 12. Angular velocity of the upper shield

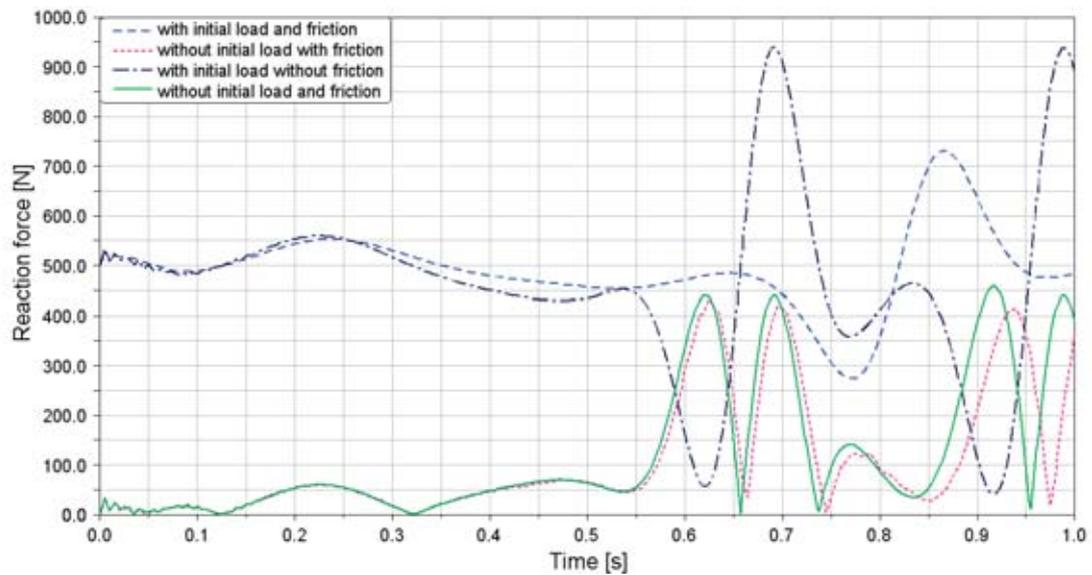


Fig. 13. Reaction forces in joint between cap and disc

## 5. Conclusions

The selected results presented in the article, and obtained with mixed FEM and MBS methods of simulation of industrial robot, may be helpful for designer of these robots to estimate their manipulations characteristics. Also it may be useful in actuators and control system synthesis. The control system can be easily considered in the model since MBS environment allows to easily implement such models prepared even in specialized external packages (like MATLAB/SIMULINK).

Besides, the way of mechanism modelling shown here, may serve as an example for modelling different robots with deformable bodies.

Mechanisms of models prepared here can be easily parameterized. That means that they can be analyzed from the performance optimization point of view. There is also possibility to verify prototype from strength point of view.

Further works on building model of mechanical system together with control system of POLYCRANK robot are in the progress.

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